B_K with two flavors of dynamical overlap fermions

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for JLQCD Collaboration

Workshop on “Domain Wall Fermions at Ten Years”

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Introduction

- Determining B_K with high precision is being within our reach.
- Calculation of B_K with two-flavor dynamical overlap fermions is underway at JLQCD.
- This is $a\ status\ report$.
  Notice that all the analysis method and results are not final ones. Especially finite volume effects are not taken into account in the following analysis..

Simulation parameters

- Gauge: Iwasaki RG (β=2.30)
  - \+ extra Wilson fermions (m_0=1+s=1.6)
    to prevent topological charge, Q, from changing
  - \+ ghosts with twisted mass μ=0.2
    to suppress unwanted UV effects due to extra Wilson fermions

$$\exp(-S_g^{\mathrm{Iwasaki}}) \Rightarrow \det\left|\frac{H_W(m_0)^2}{H_W(m_0)^2+\mu^2}\right| \exp(-S_g^{\mathrm{Iwasaki}})$$ [JLQCD, PRD74 (2006)094505]

r_0=0.49 fm ⇒ a=0.1184(12) fm ($1/a$=1.667(17) GeV)

$(L/a)^3 \mathrm{x} (T/a)=16^3 \times 32 \Rightarrow V \approx (1.9\ fm)^3$

Results are from configurations in Q=0.

Simulation parameters

- Sea and valence quarks: Overlap fermion
  - 6 sea quark masses (am_{sea}=0.015, 0.025, 0.035, 0.050, 0.070, 0.100)
    1/6 $m_s < m_q < m_s \Leftrightarrow 0.34 < m_\pi/m_\rho < 0.67$
    our lightest pion $\Leftrightarrow m_\pi \approx 293$ MeV, $m_\pi L \approx 2.8$
  - 6 valence quark masses take the same values as the sea's.
  - LMP and LMA are implemented for all valence propagators.
  - While all degenerate and non-degenerate mesons are measured,
    I focus on the degenerate mesons (m_{val1}=m_{val2}) in this talk.

Current statistics

- 10,000 trajs. are ready at all six sea quark masses.
- Measurements every 20 trajs.
- Thus, 500 x 6 = 3,000 measurements are required.
- ~60 % of total statistics is used in the following.

  (For the two heaviest m_{sea}s, statistics are very poor.)

# of data which is used here:

m_{sea}	0.015	0.025	0.035	0.050	0.070	0.100
# of data	443	439	383	295	109	191

low statistics

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# of data	443	439	383	295	109	191

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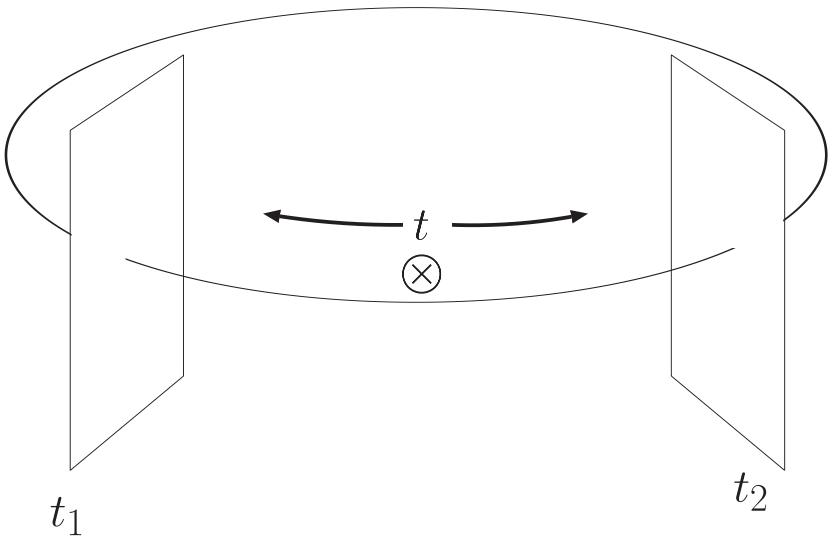
Measurement

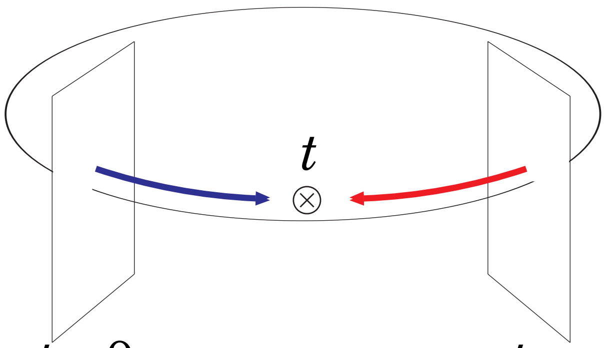


Method is very standard.

- Put wall sources at t_1 and t_2 and the four-quark operator at t.

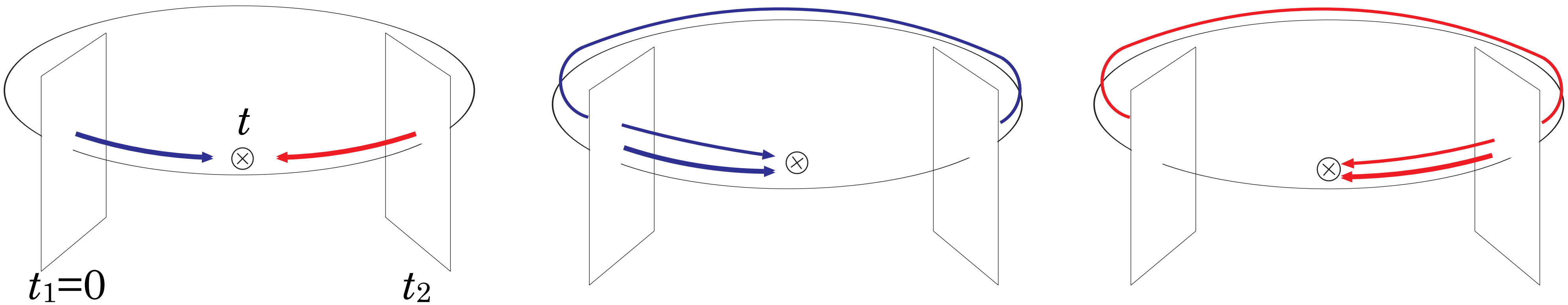
- 6 combinations of (t_1, t_2):
  (t_1, t_2)=(0,16), (8,24), (0,24), (8,0), (16,8), (24,16)
- 3-pt functions are calculated over these 6 combinations, and averaged over equivalent ones.
  $\Rightarrow$ two kinds of 3-pt with $|t_1 - t_2|$ =16 and 24

Measurement



$$
\begin{aligned}
C^{(3)}_{L_\mu L_\mu}(t,t_2,0) &= \sum_{\vec{x}} \langle 0 |\ (A_4^{\rm w}(t_2))^\dagger\ O_{L_\mu L_\mu}(t,\vec{x})\ (A_4^{\rm w}(0))^\dagger\ | 0 \rangle \\
&= \frac{V}{(2M_K)^2} Z^2_{\rm wall} \langle \bar{K} | O_{L_\mu L_\mu}\ | K \rangle e^{-M_K(N_t - t_2)}
\end{aligned}
$$

Measurement



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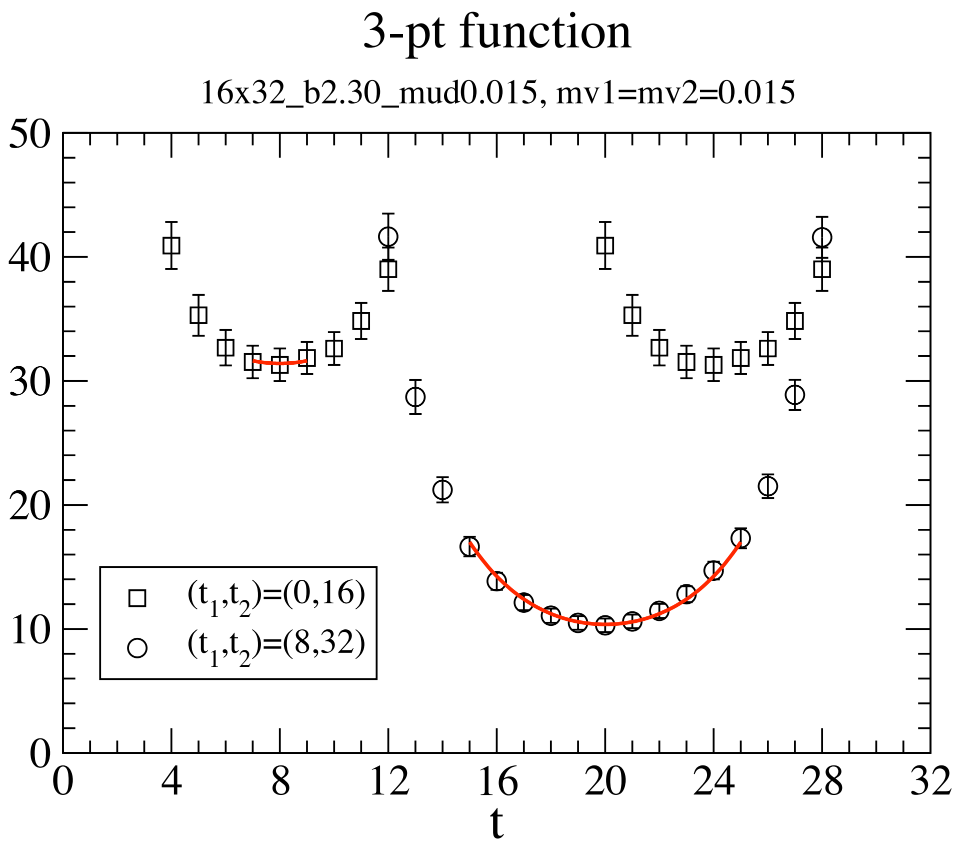
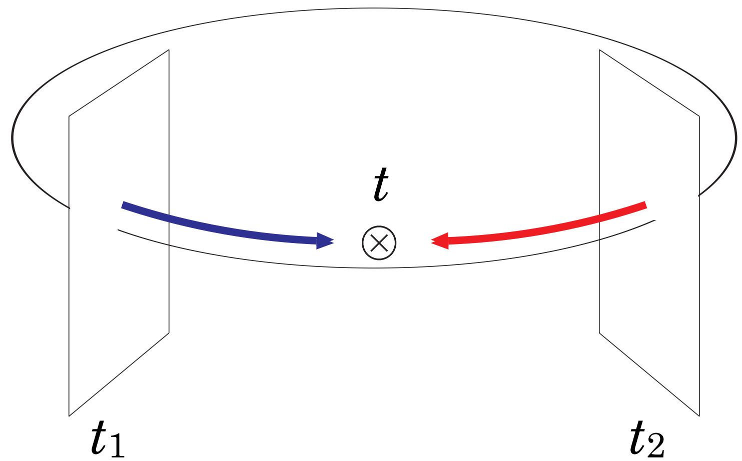
&= \frac{V}{(2M_K)^2} Z^2_{\rm wall} \langle \bar{K}|O_{L_\mu L_\mu}\ |K\rangle e^{-M_K(N_t-t_2)} \\
&\quad + \frac{V}{(2M_K+\Delta_K)(2M_K)} Z^2_{\rm wall} \langle 0|O_{L_\mu L_\mu}\ |K,K\rangle e^{-M_K N_t - \Delta_K (N_t - t_2)/2} \\
&\quad \times \cosh\left[(M_K + \Delta_K/2)(2t - t_2 - N_t) \right],
\end{aligned}
$$

where Δ_K is a mass shift in KK system.

Fit of 3-pt function





Simultaneous fit of two 3-pt functions allows for a clean extraction of the mixing amplitude.

$$
\begin{aligned}
C^{(3)}_{L_\mu L_\mu}(t, t_2, 0) &= \sum_{\vec{x}} \langle 0 |\ (A_4^{\rm w}(t_2))^\dagger\ O_{L_\mu L_\mu}(t, \vec{x})\ (A_4^{\rm w}(0))^\dagger\ | 0 \rangle \\
&= \frac{V}{(2M_K)^2} Z^2_{\rm wall} \langle \bar{K} | O_{L_\mu L_\mu}\ | K \rangle e^{-M_K (N_t - t_2)} \\
&\quad + \frac{V}{(2M_K + \Delta_K)(2M_K)} Z^2_{\rm wall} \langle 0 | O_{L_\mu L_\mu}\ | K, K \rangle e^{-M_K\, N_t - \Delta_K (N_t - t_2)/2} \\
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\end{aligned}
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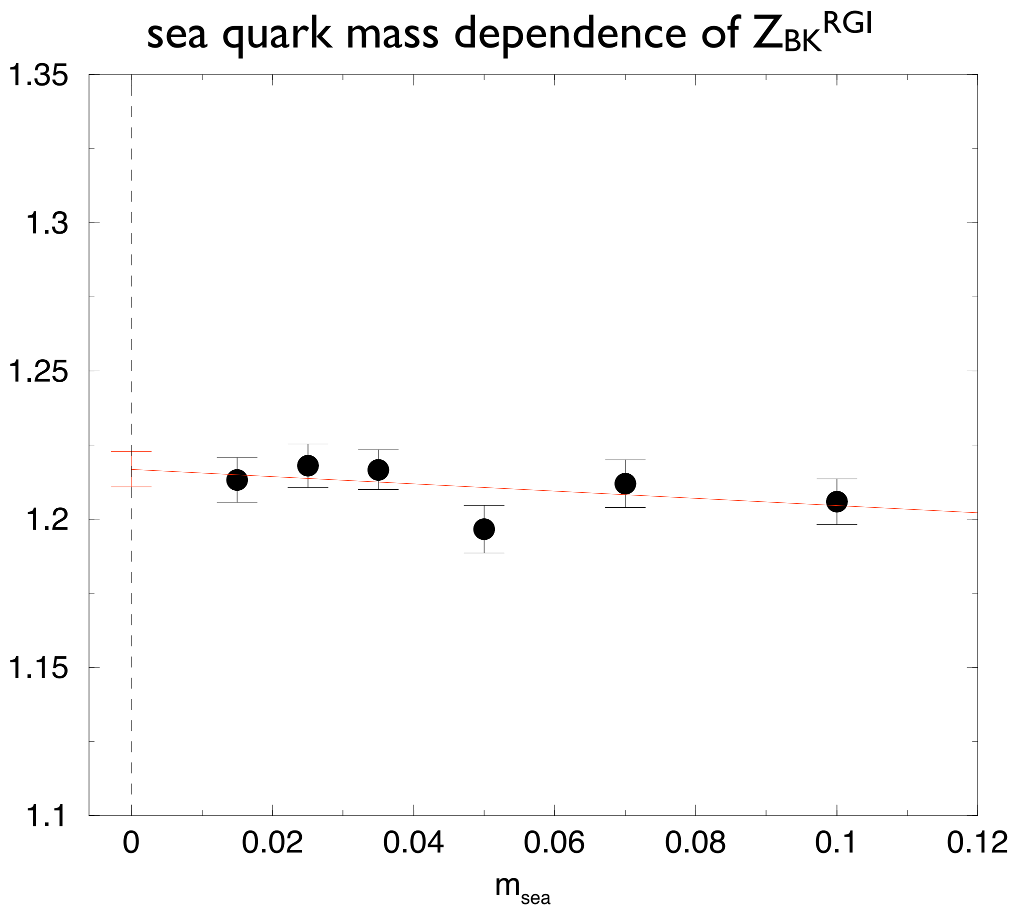
NPR with RI-MOM



Preliminary result :

$$Z_{B_K}^{\mathrm{RGI}} = 1.217(6)$$

$$Z_{B_K}^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.862(4)$$

In the following, focus on $B_K(2\ \mathrm{GeV})$.

Previous dynamical simulations

N_f=3

- RBC and UKQCD, hep-ph/0702042
  - DWF, a~0.12 fm
- HPQCD and UKQCD, PRD73(2006)114502
  - improved staggered, a~0.125 fm
- Kim, Bae, Lee and Sharpe, Pos, LAT2006(2006)086
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N_f=2

- SPQcdR, Pos, LAT2005(2005)365
  - Wilson, a~0.063 fm
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  - clover, a~0.1 fm
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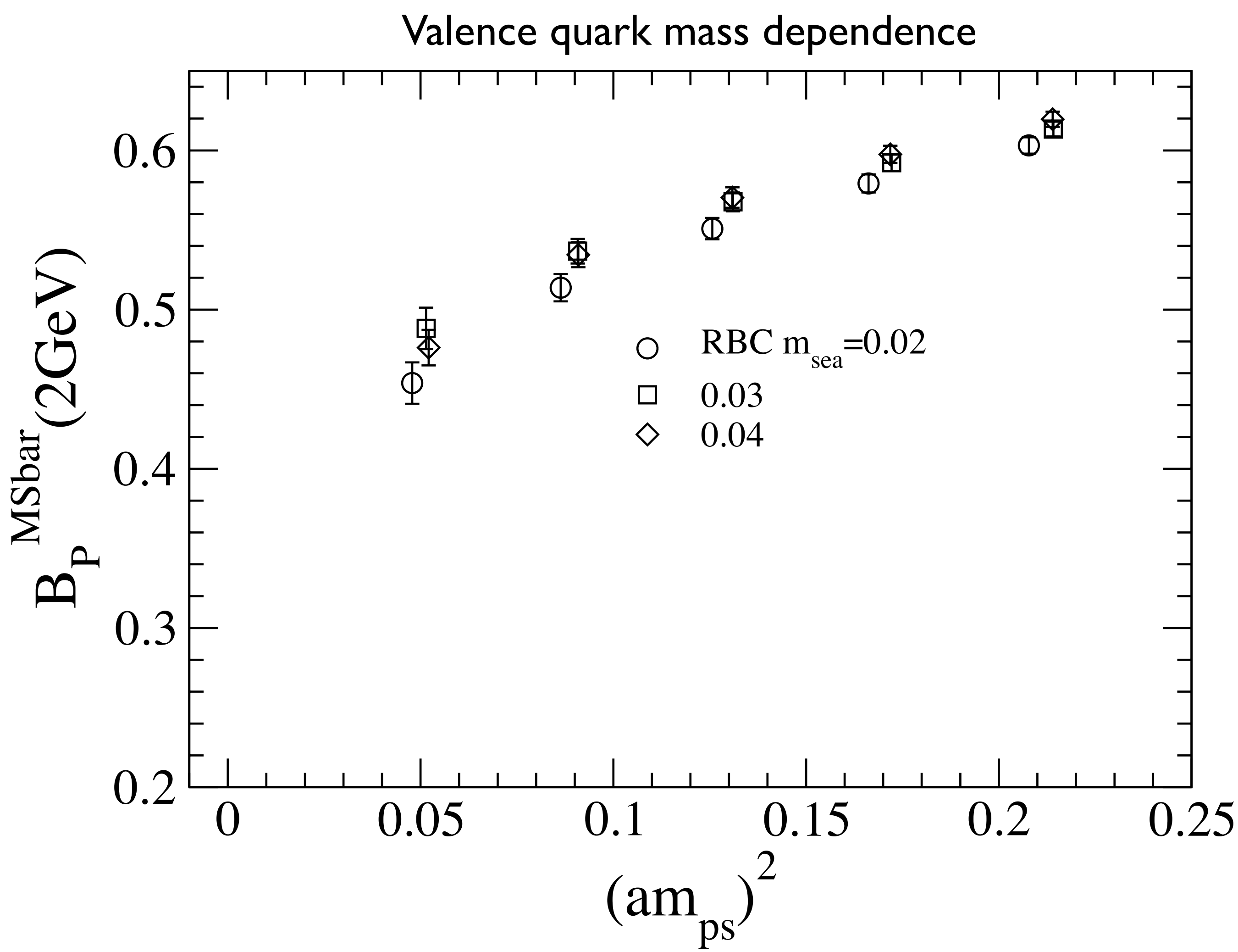
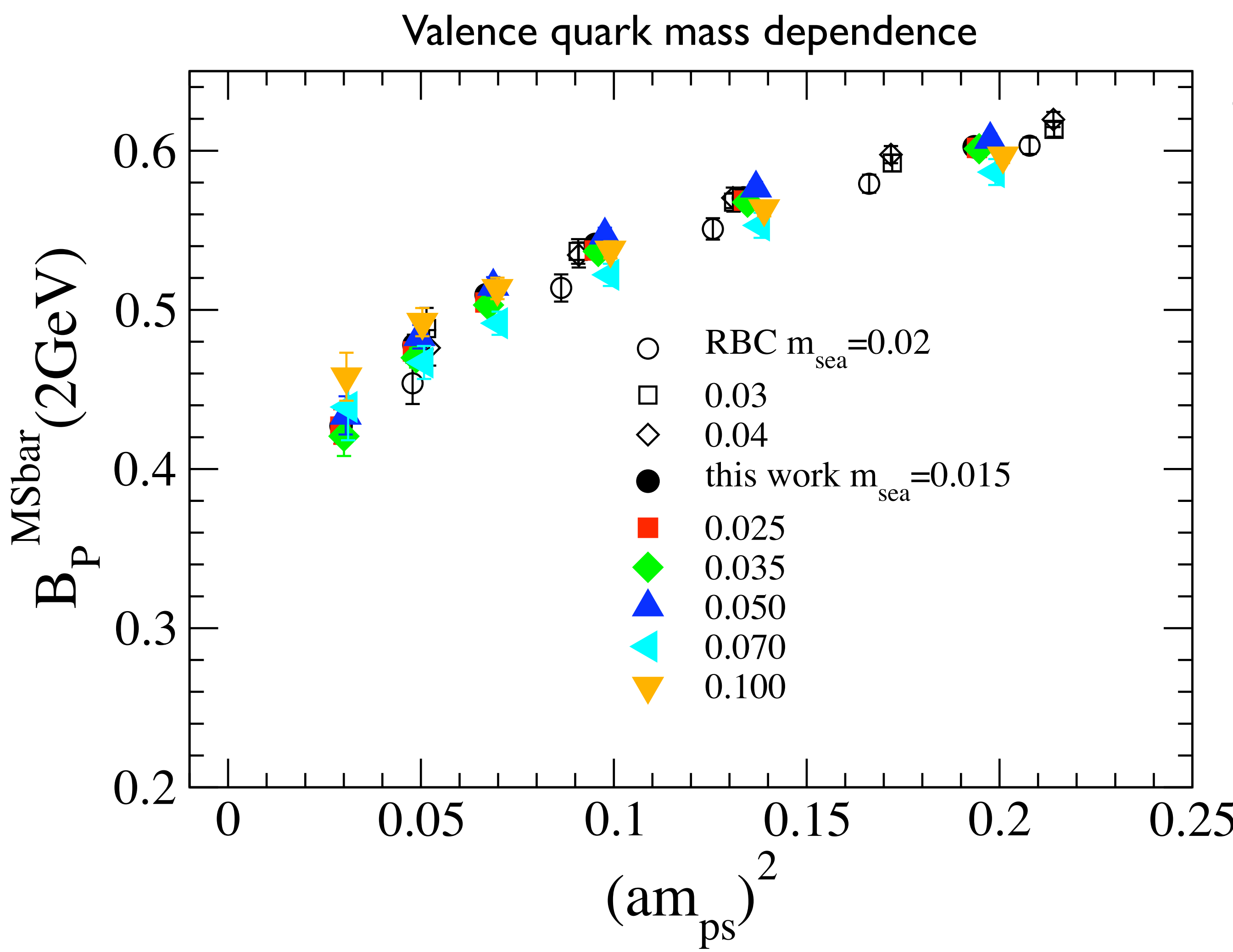
Our result will be frequently compared with this.

Results at first glance

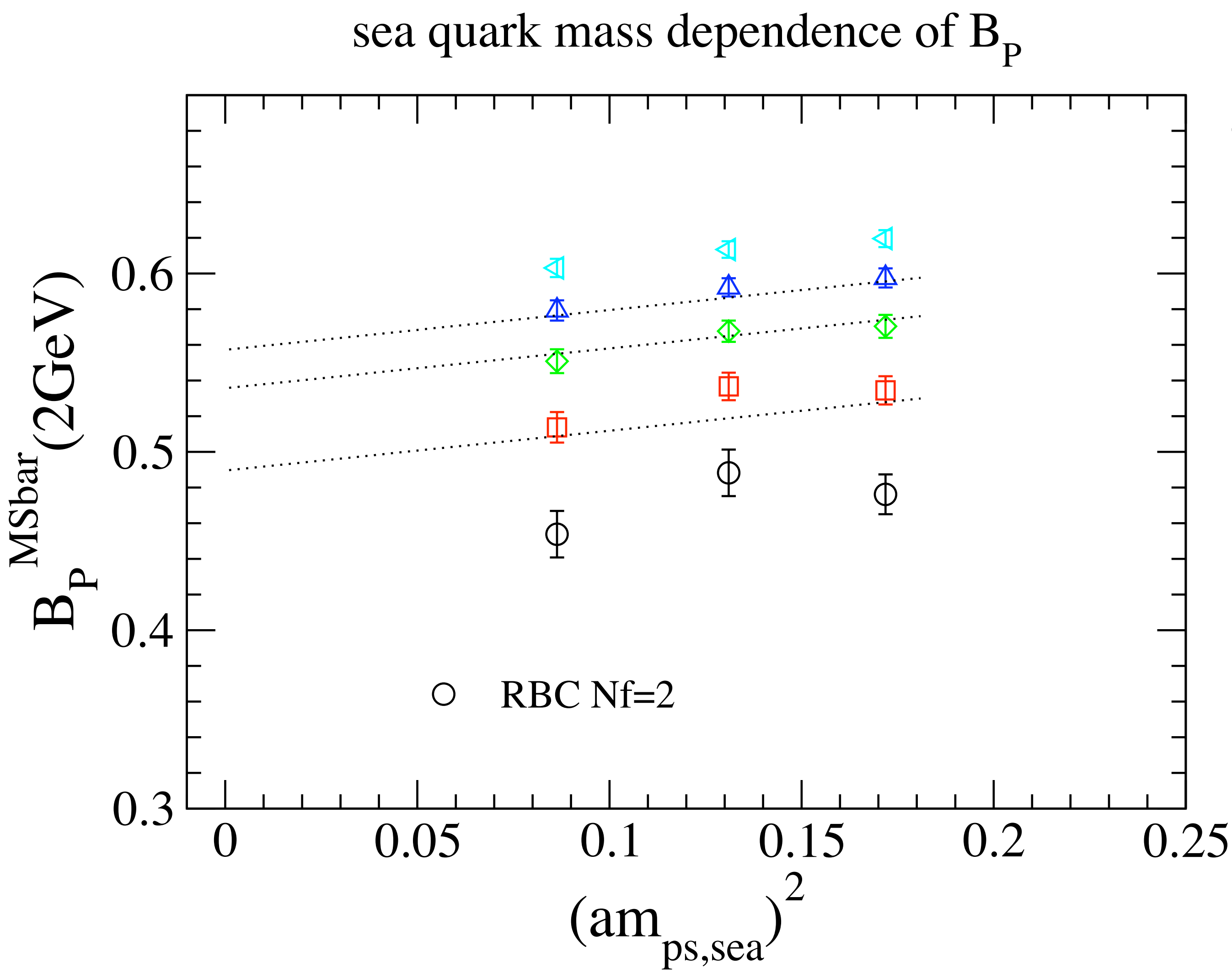


Results at first glance



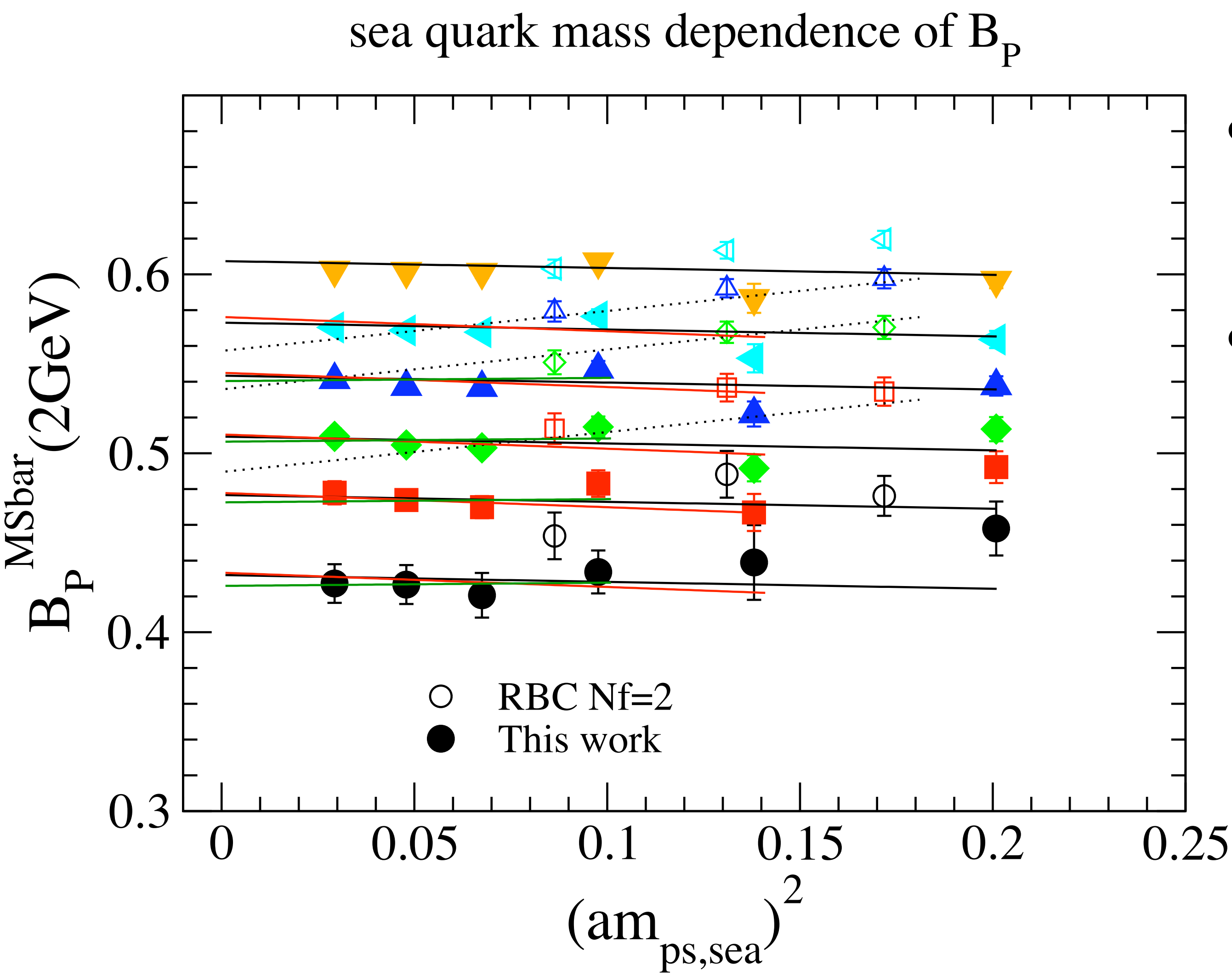
- Consistent with the RBC's result, and the difference looks minor.

Sea quark mass dependence



- RBC observed a small but non-zero positive slope in m_{sea} dependence.

Sea quark mass dependence



- RBC observed a small but non-zero positive slope in m_{sea} dependence.
- Our data do not show clear slope.

Test with NLO ChPT

- NLO PQChPT formula for B_K with degenerate $m_{\rm val}$:

Golterman and Leung, PRD57(1998)5703

$$B_P = B_P^\chi \left[1 - \frac{2m_{\rm ps}^2}{(4\pi f)^2} \left\{ 3 \ln \left(\frac{m_{\rm ps}^2}{\mu^2} \right) + 1 \right\} \right] + C_{\rm val}\, m_{\rm ps}^2 + C_{\rm sea}\, m_{\rm ps,sea}^2$$

  - four free parameters

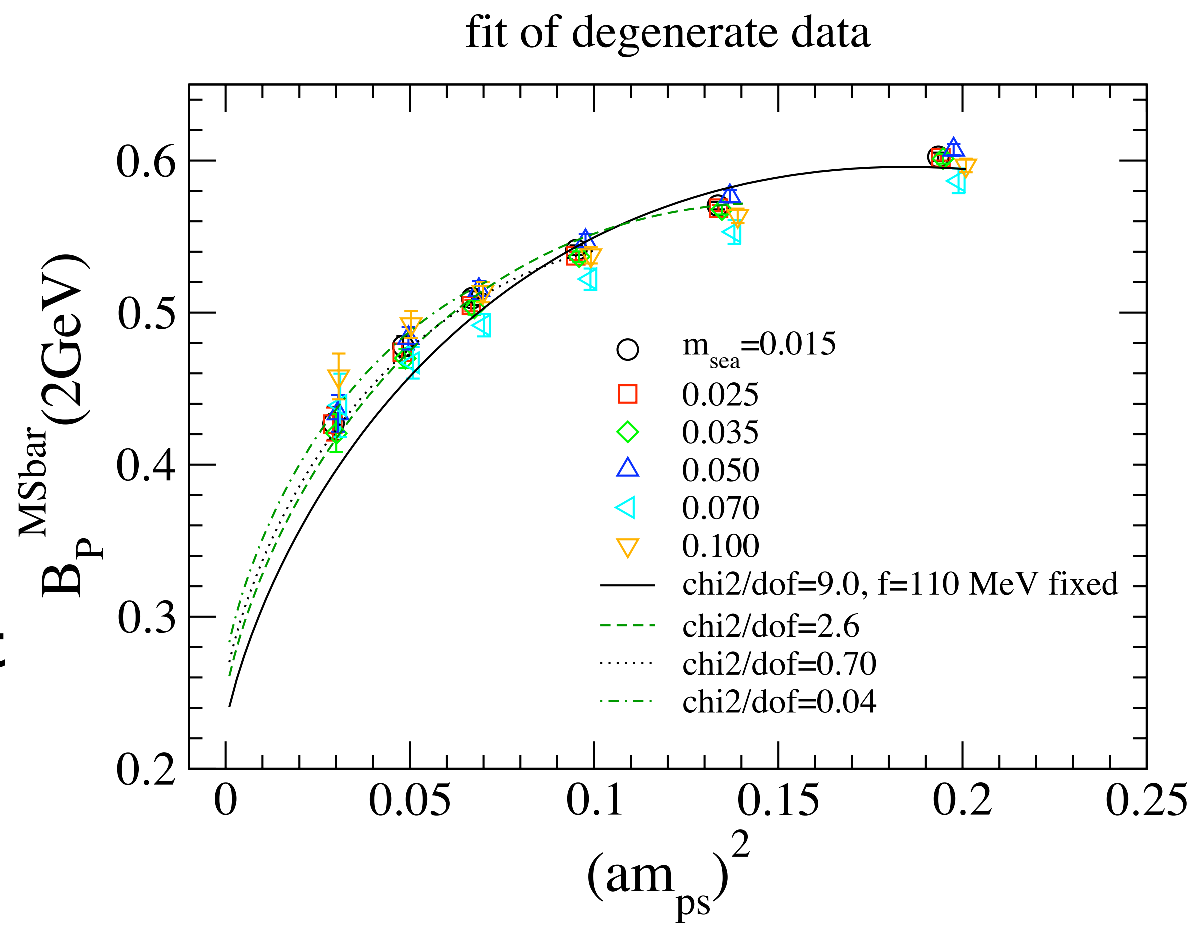
- How to test
  - Fit the data to the above formula
    (i) with fixed f, (hence three free parameter in this case)
    (ii) with free f.
  - Changing fit range to find the fit range well described by the NLO PQChPT formula.

Test with NLO ChPT

(i) with fixed f

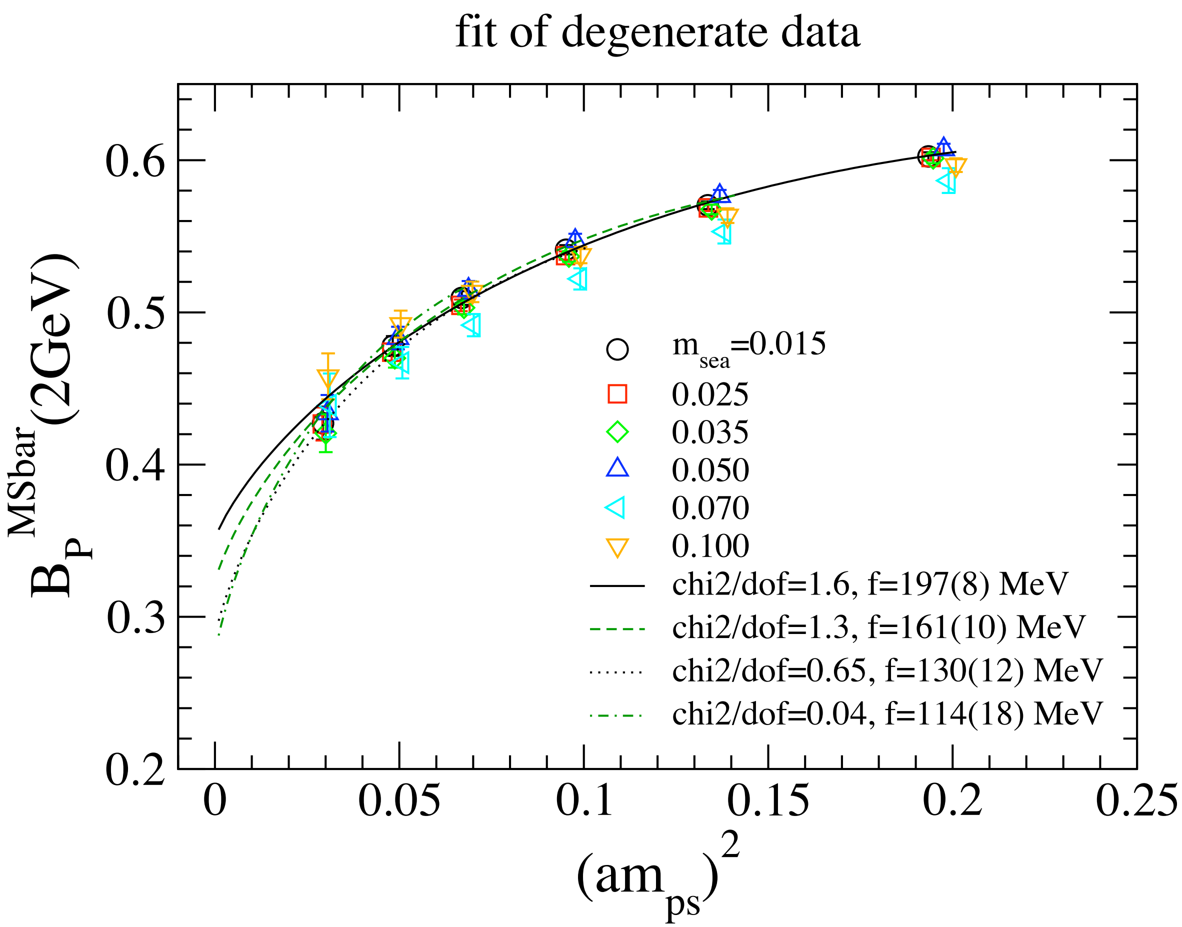
- $f = 110$ MeV as an example
- χ^2/dof reduces as the fit range becomes smaller.
- When the fit range is $[0, m_s/2]$ or smaller fit becomes reasonable.

Somewhat confident of that our three or four lightest quarks are well inside the NLO ChPT regime.



Test with NLO ChPT

(ii) with free f



- While χ^2/dof is reasonable for all fit ranges, f largely depends on them.
- When the fit range is $[0, m_s/2]$ or smaller, f takes a reasonable value.

Surely confident of that our three or four lightest quarks are well inside the NLO ChPT regime.

Introducing higher order terms

How are the data *outside* the ChPT regime described?

1. add O(p^4) term to NLO naively: (5 free parameters)

$$B_P = B_P^{\chi}\left[1 - \frac{2m_{\rm ps}^2}{(4\pi f)^2}\left\{3\ln\left(\frac{m_{\rm ps}^2}{\mu^2}\right)+1\right\}\right] + C_{\rm val}\, m_{\rm ps}^2 + C_{\rm sea}\, m_{\rm ps,sea}^2 + \underline{C_{\rm val,2}\,(m_{\rm ps}^2)^2}$$

2. add your favorite function to NLO but keeping the previous fit results:

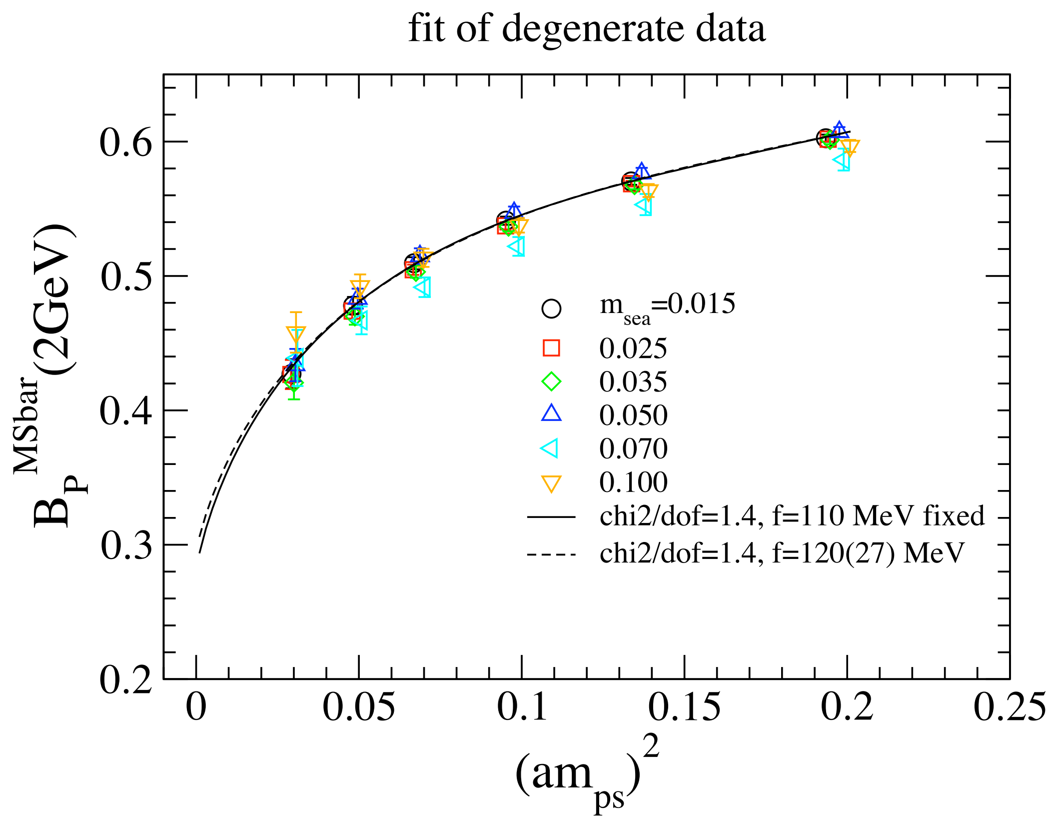
$$B_P = B_P^{\chi}\left[1 - \frac{2m_{\rm ps}^2}{(4\pi f^2}\left\{3\ln\left(\frac{m_{\rm ps}^2}{\mu^2}\right)+1\right\}\right] + C_{\rm val}\, m_{\rm ps}^2 + C_{\rm sea}\, m_{\rm ps,sea}^2$$

$$\underline{+ \quad \theta(m_{\rm ps}^2-\mu^2)\, C_{\rm val,2}\,(m_{\rm ps}^2-\mu^2)^2}$$ (1 free parameters)

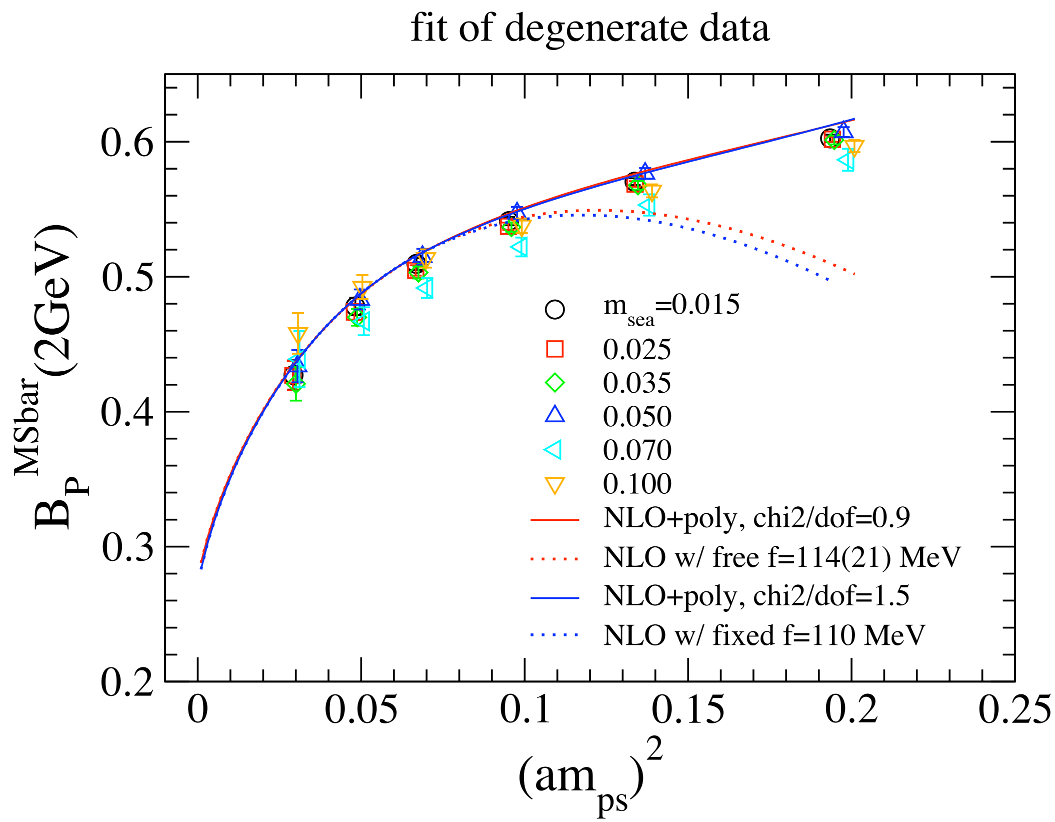
Both fits are done (i) with fixed f and (ii) with free f.

Introducing higher order terms

- naive: (4 or 5 free parameters)



- non-naive: (1 free parameters)



In both cases, the whole data are well interpolated.

Preliminary result for B_K

Interpolate to physical m_K using the NNLO-like functions to obtain B_K at m_{sea}=m_{ud} and m_{val1}=m_{val2}=$m_s/2$ as

$$B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = \begin{cases} 0.534(3) & \text{naive NNLO w/ free } f \\ 0.540(10) & \text{non-naive w/ free } f \end{cases}$$

$$\text{(statistical error only)}$$

$$B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.509(18) \quad \text{[RBC (incl. systematic error)]}$$

Summary

- Calculation of B_K is now in progress.
- Preliminary result looks promising.
- Our three or four lightest quarks are well inside the NLO ChPT regime.
- To do
  - include non-degenerate mesons,
  - study topological charge dependence numerically,
  - clarify finite volume effects using ChPT with FV,
  - estimate systematic errors.

Effects of fixed topology

According to the argument in large N_c,
[see Brower, Chandrasekharan, Negele and Wiese, PLB560(2003)64],

it is naively expected that such an effect appears only at NLO of ChPT through

$$m_\pi(Q) \approx m_\pi^{\rm phys}\left[1 - \frac{1}{4N_f m\Sigma V_4}\left(1 - \frac{N_f Q^2}{m\Sigma V_4}\right)\right],$$

where V_4 is four dim. space-time volume, $\Sigma = \langle \bar{q}q \rangle$

hence we expect the relatively small effect on B_K compared to that on m_π itself.